# John Nash and the analysis of rational behavior (Greek translation)

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# John Nash and the Analysis of Rational Behavior*

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## 1. INTRODUCTION

When playing a game, we ask ourselves "what should I do?" When observing a game being played, we ask ourselves "what will be the outcome of the game?" Both questions are difficult to answer as the answers will depend on the players' skills and personalities, their emotions, their motivation, their determinateness, etc. To make scientific progress, it, hence, seems appropriate to first eliminate these "frictions" and rather try to answer a different question first, one that also provides a benchmark to evaluate the importance of these "real world frictions". John Nash was the first to systematically address this more fundamental question of how a game will be played by "rational" players. Nash's answer paved the way for the unified methodology that we find in the social sciences today.

The axiomatic approach that proved so powerful in the hands of Nash, had been pioneered in economics by John von Neumann and Oskar Morgenstern. In their great book (Von Neumann and Morgenstern, 1944) they outline, in Section 17.3, the requirements that a theory of rational behavior should satisfy. They, however, develop a theory consistent with these requirements only for two special cases, viz. the 1-person case and the 2-person zero-sum case. It is still a mystery today why they didn't pursue the argument more generally, certainly given Morgenstern's earlier writings on the topic of equilibrium (Morgenstern, 1935). From today's perspective, von Neumann and Morgenstern left three important building blocks, viz. the insight that games could serve as useful general models for social conflict situations the fundamental concept of strategy that allows the drastic simplification to games in normal form, and the result that consistent preferences could be represented by a utility function that is linear in the probability distribution over outcomes. Nash added the fourth fundamental building block, the equilibrium notion that specifies how the institutions (game rules), strategies and player preferences interact to produce the overall outcome. The four together provide a unified structure for analysing all situations of social conflict and cooperation.

In this essay, I briefly describe Nash's pathbreaking contributions to economics. I discuss both his contributions to non-cooperative game theory, as well as those to cooperative bargaining theory. Nash's third fundamental contribution links these two in the so-called Nash program. I conclude with some observations on Nash's work in experimental economics, and on the role that concepts developed by Nash play in applied economics today.

## 2. RATIONAL EQUILIBRIUM

An interactive decision situation, or game, is one in which various actors (players) are involved that jointly determine the outcome and in which each tries to obtain that outcome that is most favourable to him. Imagine the situation is one of complete information, meaning that each player is fully informed about the other players' preferences and possible strategies. Assuming that, for each of the players, there is a unique rational way to play the game, one may as well assume that rational players know this "best" way to play. A rational player will then know what his opponents will do in the game, which strategies they will play, and he will be willing to act in conformity with the rational theory if it is indeed best for him, i.e. if his strategy is a best response to the strategies of the others.

John Nash outlined the above line of logic in the PhD-thesis[1] that he submitted to the Department of Mathematics at Princeton University in May, 1950 (Nash, 1950a). Formally, define an *n*-person game as a tuple $<S_1,...,S_n,u_1,...,u_n>$ where $S_i$ is the (finite) set of pure strategies of player *i* and $u_i: S \to \mathbb{R}$ is player *i*'s utility function, defined on the set of strategy profiles $S = \times_i S_i$. Writing $\Sigma_i$ for the set of mixed strategies of player *i* (probability distributions on $S_i$) and assuming that players' preferences satisfy the consistency assumption discussed by Von Neumann and Morgenstern, the utility functions can be multi-linearly extended to $\Sigma = \times_i \Sigma_i$. Expecting $\sigma \in \Sigma$ to be played, each player *i* is tempted to deviate to a strategy $\tau_i$ that maximizes *i*'s expected payoff given that the opponents play their parts of $\sigma$. A strategy profile $\sigma$ is said to be a (Nash) equilibrium if no single player has a profitable deviation, i.e. if $\sigma$ is a best response against itself.

The main result of Nash's thesis, the existence of at least one equilibrium in every finite game, was announced in the Proceedings of the National Academy of Sciences in 1950 (Nash, 1950b). The proof amounts to noting that the map that assigns to each $\sigma$ to set of all best responses to $\sigma$ satisfies the conditions of the Kakutani Fixed Point Theorem. This technique of proof, which was then introduced in the literature, has become standard in the area of mathematical economics and game theory. In Nash's thesis, an elegant, more elementary proof is given that relies on Brouwer's fixed point theorem. This proof was reproduced in an article in the Annals of Mathematics in 1951 (Nash, 1951). In essence, that article reproduces the entire PhD-thesis, apart from the Section "Motivation and Interpretation". Looking back we can say that the decision to cut out that Section was unfortunate as it had the effect of Nash's equilibrium concept being misunderstood and incorrectly interpreted for too long a time. As a consequence, the "game theoretic revolution in economics" was delayed for some time.

The PhD-thesis and the 1951 article also contain several examples to illustrate the equilibrium concept and to show that, while being an equilibrium is necessary for being a rational outcome, the condition is

[1] The thesis also gives a second interpretation of equilibrium points as stable rest point of learning processes in games played by populations of players with limited information. Much recent work in game theory relies on this second interpretation.

not sufficient. Noteworthy is the fact that the "Prisoner's Dilemma", that played such a great role in the development of the social sciences, is already given as an example. Of course, the name of the game was coined only later, by Nash's PhD-supervisor, A.W. Tucker. Another example is of a game with an unstable equilibrium and later an entire literature on "equilibrium refinements" developed that tried to eliminate such unstable equilibria (See Van Damme, 1991). Another example in the thesis has two stable (strict) equilibria, *(a,á)* and *(b,â)*, and it is accompanied by the intriguing sentence "However, empirical tests show a tendency toward *(a,á)*". The structure of this example is such that the player that goes for *(b,â)* looses a rather large amount in case of miscoordination, hence, the equilibrium *(a,á)* is safer and players may indeed be expected to coordinate on this safer equilibrium. Using modern terminology, we say that *(a,á)* is the "risk dominant equilibrium" of the game (Harsanyi and Selten, 1988).

Nash's original ideas to eliminate unstable or dominated equilibria as candidate solutions were further developed by John Harsanyi and Reinhard Selten, who shared with him the Nobel Prize in Economics in 1994. Most importantly, Harsanyi extended Von Neumann's game model so as to be able to include incomplete information and he showed how Nash's equilibrium concept could also be applied to that more general model (Harsanyi, 1967-8). Selten (1975) initiated the refinements literature that discussed how equilibria that rely on incredible threats can be eliminated. Together, Harsanyi and Selten developed their general theory of equilibrium selection that has the concept of "risk dominance" as an essential building block and that aims at generalizing the solution that Nash provided to bargaining games.

## 3. **BARGAINING**

Nash's first contribution to bargaining theory (Nash, 1950c) was written while he was still an undergraduate student. Again, the problem is idealized by assuming that the bargainers are rational and have full information about each other's preferences. The aim is to provide a unique solution, at least in value terms, so as to enable each individual to determine what it is worth to be able to participate in the bargaining. The axiomatic approach that Nash proposes is new. It consists of making a few general assumptions that the bargaining outcome should satisfy and showing that these assumptions actually determine the outcome uniquely.

Nash restricts himself to 2-player bargaining problems in which "disagreement" is a well-defined outcome. (Technically, the bargaining is with "fixed threats".) Rational bargainers are assumed to have Von Neumann-Morgenstern utility functions and a representation may be chosen that assigns to disagreement the utility zero for each player. A bargaining problem now gives rise to a set *C* of utility pairs $(u_1, u_2)$, with $(0,0) \in C$. The linearity of the VNM-functions implies that $C$ is convex and, if the underlying set of alternatives is finite, $C$ will be compact as well. One axiom now is that the solution of the problem, denoted $c(C)$, only depends on the set $C$. Of course, two sets that represent the same utility functions should have the same solutions. Secondly, if $C$ is symmetric, $c(C)$ should be symmetric. Thirdly, as rational bargainers will exploit all gains from trade, $c(C)$ should be Pareto optimal in $C$. To these very natural assumptions, Nash adds the powerful axiom of "independence of irrelevant alternatives": if $C \subseteq D$ and $c(D) \in C$, then $c(C) = c(D)$. One may think of the solution as beating every alternative in a pairwise contest. Clearly, if an alternative beats all others in a certain set, then it will also beat all those in each subsets. Variants of this axiom have been used, and have played an important role, in other parts of the economic literature, such as social choice theory (Arrow's impossibility theorem).

Nash proves that, for the domain under consideration, the axioms determine the solution uniquely. Specifically, $c(C)$ is that point in $C \cap \mathbb{R}^2$ where the product of the utilities $u_1 u_2$ reaches its maximum. The proof first notes that the utility representation may be chosen to ensure that $c(C) = (1,1)$ and it then applies the IIA-axiom to $C$ and the symmetric set $D = \{(u_1,u_2): u_1 + u_2 \leq 2\}$.

## 4. **THE NASH PROGRAM**

In his 1953 paper, Nash extends his bargaining theory to 2-person games with variable threats. Given is a 2-person game $<S_1,S_2,u_1,u_2>$ and players negotiate which correlated strategy to play; if negotiations break down, players choose their strategies independently. Because players are supposed to be able to discuss the situation and to agree on a joint plan of action, Nash refers to the situation as a cooperative game. The paper gives a solution and derives this solution in two independent, complementary ways. The first approach is axiomatic. In addition to the axioms from the earlier paper, two new axioms stipulate how the solution should vary with changes in the strategy sets. These axioms are completely natural, stating that a player cannot gain by having fewer threats available and that one is not really hurt as long as remains an optimal threat available. One sees that these axioms allow reduction of the present problem to one with fixed threats.

The second approach amounts to modelling the bargaining process as a non-cooperative game, hence, the cooperative game is reduced to a non-cooperative one, that can then be analysed using the equilibrium concept. Nash is aware that real life bargaining involves various intracies and details in the rules, hence, that skillful modelling is required. He writes "Of course, one cannot represent all possible bargaining devices as moves in the non-cooperative game. The negotiation process must be formalized and restricted, but in such a way that each participant is still able to utility all the essential strengths of his position" (p. 129). He then formulates a 2-stage negotiation process. In the first stage, players choose the "threat strategies" that they will be committed to use if negotiations break down; in the second stage, players, knowing the threats, simultaneously state utility demands; if these demands can be met, payoffs are accordingly, otherwise the threat strategies are implemented.

It is easy to see that the second stage demand game typically has multiple Nash equilibria. For example, if players are bargaining over the set $C = \{(u_1,u_2)\in\mathbb{R}^2;\ u_1 + u_2 \leq 1\}$, then any non-negative demand vector $(d_1, d_2)$ with $d_1 + d_2 = 1$ constitutes an equilibrium. However, Nash notes that these equilibria have different stability properties and he argues that one equilibrium is most stable and especially distinguished. Specifically, he imagines that players will be somewhat uncertain about which combinations of demands are feasible and he shows that only one equilibrium survives when this uncertainty is taken into account. In fact, only the equilibrium in which the product of the utilities is maximized, that has been identified by the axiomatic approach, is robust in this sense. Having solved the second stage in this way, the first stage reduces to a strictly competitive game that can be solved by the equilibrium concept. Robustness tests of the type that Nash introduced in this paper have played an important role in the refinements literature that was already referred to above.

Nash's non-cooperative bargaining model is just one model of the bargaining process and, perhaps, it is not the most natural one. In addition, Nash's game is plagued by multiplicity of equilibria. Even though an ingenious and seminal argument could be used to obtain uniqueness, one might expect the methodology to be not universally applicable. These technicalities, however, should not distract from the most important aspect of Nash's 1953 paper: the suggestion to analyse cooperative problems by means of non-cooperative models and the demonstration that the proposed method of analysis is feasible. Later, other game theorists have followed up Nash's suggestion and they have come up with natural non-cooperative models that do not suffer from these drawbacks. One example is Rubinstein's (1983) bargaining model, in which players alternate in making offers until agreement is reached, or until a chance event exogenously determines breakdown. Quite remarkably, this natural bargaining procedure again produces the solution that was first identified by Nash.

## 5. **CONCLUSION**

In his paper "What is game theory trying to accomplish?", Bob Aumann has forcefully argued that a game theoretic solution concept should be primarily judged by the insight that it yields into the workings of the social processes to which it is applied. Aumann (1987, p. 48) also writes that on this score "Nash equilibrium is without a doubt the most 'successful' - i.e., widely used and applied- solution concept of game theory. It touches almost every area of economic theory, as well as social choice, politics and many other areas of application". Aumann puts forward the view that comprehension is the basic aim of science and he states that "predictions are an excellent means of testing our comprehension, and once we have the comprehension, applications are inevitable. (p. 29).

We may follow up on Aumann's general remarks by giving two concrete examples from the recent policy context. Throughout the world, auctions are increasingly being used to transfer resources from the government's hands into more efficient and productive ownership. In the design of these auctions, game theorists have played an important role and their advise has in part been based on Nash equilibrium analysis of simplified, related game models. Secondly, the European Commission has recently blocked a merger between the truck producers Scania and Volvo. Merger analysis is increasingly based on quantitative techniques in which one estimates product differentiation price competition models and compares the Nash equilibrium outcomes before and after the merger, deciding that the merger will be blocked if prices will rise to such an extent as to hurt consumer welfare. These are just two examples of applied work based on Nash equilibrium analysis,  many more could be added.

As Nash stressed in his papers, his work is built on the simplifying assumption that the players are highly intelligent and rational individuals. Real human beings may not be able or willing to be that rational and a question remains about the contexts in which the rational theory provides a relevant benchmark for boundedly rational behavior. I close by noting that Nash himself already called for empirical investigation using the experimental approach and that also his experimental work may be a source of inspiration for many (See Kalisch et al. (1954)).

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